استخدام طريقه ادوميان التحليلية لإيجاد حل لمعادله بيسل

USE ADOMIAN DECOMPOSITION METHOD TO FIND SOLUTION FOR BESSEL's EQUATION

سماح سعيد سالم سعيد[1]

samah saeed salim saeed

أ.م. د. يحيي قائد حسن[2]

Asso.Prof.Dr :Yahya Qaid Hasan

https://doi.org/10.54582/TSJ.2.2.21

(1) باحثة.

عنوان المراسلة : : samah2414@gmail.com

(2)أستاذ مشارك في جامعة إقليم سبأ.

عنوان المراسلة : yahya217@yahoo.com

الملخص

في هذه الورقة ، قدمت مؤثر تفاضلي جديد هو

$$L(.) = x\, j_{\frac{1}{2}}(x) \frac{d}{dx}\, x^{-1} j_{\frac{1}{2}}(x)^{-2} \frac{d}{dx}\, j_{\frac{1}{2}}(x)(.)$$

لحل معادله بيسل $y'' + \frac{1}{x} y' + \left(1 - \frac{1}{4\,x^2}\right) y = G(x, y)$

باستخدام طريقه ادوميان التحليلية و تم إعطاء مثالين لفهم كيفيه استخدام المؤثر في حل المعادلة.

الكلمات المفتاحية: معادله بيسل، طريقه ادوميان التحليلية، معادلات تفاضليه من الرتبة الثانية .

https://doi.org/10.54582/TSJ.2.2.21

Abstract

The paper present new differential operator

$$L(.) = x\, j_{\frac{1}{2}}(x)\ \frac{d}{dx}\ x^{-1}\left(j_{\frac{1}{2}}(x)\right)^{-2}\ \frac{d}{dx}\ j_{\frac{1}{2}}(x)(.)$$ **to solve Bessel's equation**

$$y'' + \frac{1}{x}y' + \left(1 - \frac{1}{4\,x^2}\right)y = G(x, y)$$

by using Adomian decomposition method , the method has been explained in research and give two examples to understand how to use the operator in solving the equation.

Keywords : Bessel 's equation, Adomian decomposition method, second ordered ordinary differential equations.

1 Introduction:

The laws of physics are generally written as differential equations. Therefore, all of science and engineering use differential equations to some degree. You can think of mathematics as the language of science, and differential equations are one of the most important part of this language as far as science and engineering are concerned [6]. The ordinary differential equation

$$x^2y'' + xy' + (x^2 - p^2)y = 0,$$

is known as Bessel'sequation of orded p [9] . Bessel functions are solutions of Bessel's equation[7], and we can solve it by Laplace equation in polar coordinates by the needed of separation of variables [8]. Also, to solve the Bessel's equation we apply the Frobenius method by assuming a series solution, but in this work we use the Adomian Decompostion Method. The Adomain Decomposition Method is considered as one of the most effective methods in finding convergent as well as complete solution. In the year 1980s [3, 4], the Adomian decomposition method (ADM) appeared by the American scientist Geoge. This method solved many equations that the traditional methods were unable to solve. Studied in this method [5, 10] showed the efficiency and effectiveness of this method in finding approximate solution of different types of equations. Our goal in this work is to find approximate solutions for this type of equations using a modified method with boundary value problems. we introduced a new operator that could solve like these equations. Aboundary value problem in one dimension is an odrinary differential equation together with conditions involvingvalues of the solution and its derivatives at two or more points. The number of conditions imposed is equal to the order of the differential equation [1].

2 Adomian Decompostion Method:

The Bessel's Equation of order half.

$$y'' + \frac{1}{x}y' + (1 - \frac{1}{4x^2})y = G(x,y) \tag{1}$$

The equation (1)can be formulated as:

$$Ly = G(x,y),$$

so that the differential operator is

$$L(.) = xJ_{\frac{1}{2}}(x)\frac{d}{dx}x^{-1}J_{\frac{1}{2}}(x)^{-2}\frac{d}{dx}J_{\frac{1}{2}}(x)(.),$$

and the invers operator is as follows :

$$L^{-1}(.) = J_{\frac{1}{2}}(x)\int_0^x x^{-1}J_{\frac{1}{2}}(x)^{-2}\int_a^x xJ_{\frac{1}{2}}(x)(.)dxdx,$$

when we take L^{-1} to both sides we get

$$y = \phi(x) + L^{-1}G(x,y) + L^{-1}U(x,y),$$

such that $G(x,y)$ is an unknown function , $U(x,y)$ is a non linear part and

$$\phi(x) = aJ_{\frac{1}{2}}(x)tan(x)(J_{\frac{1}{2}}(a))y'(a) - aJ_{\frac{1}{2}}(x)tan(x)(J'_{\frac{1}{2}}(a))y(a).$$

The Adomian Decomposition Method gives solution of $U(x,y)$ by infinite series

$$y(x) = \sum_0^\infty y_n(x), \tag{2}$$

and

$$U(x,y) = \sum_0^\infty A_n(x). \tag{3}$$

The Adomian polynomials are

$$A_0 = U(y_0),$$

$$A_1 = y_1U'(y_0),$$

.... now we get

$$\sum_0^\infty y_n(x) = \phi(x) + L^{-1}\sum_0^\infty A_n(x),$$

the y_n can by found as following :

$$y_0 = \phi(x) + G(x,y),$$

$$y_{n+1} = L^{-1}A_n, n \geq 0,$$

then

$$y_0 = \phi(x) + G(x,y),$$

$$y_1 = L^{-1}A_0,$$

$$y_2 = L^{-1}A_1,$$

$$y_3 = L^{-1}A_2.$$

[2, 5]

Example(1):

Consider the equation

$$y'' + \frac{1}{x}y' + (1 - \frac{1}{4x^2})y = \sqrt(x) - x + y^2 \qquad (4)$$

the equation (4)is re-written as

$$Ly = \sqrt(x) - x + y^2,$$

we use L^{-1} fot it and get

$$y = \phi(x) + L^{-1}(\sqrt(x) - x) + L^{-1}(y^2),$$

and

$$\phi(x) = e\frac{sin(x)}{\sqrt(x)}(\frac{3}{2}cos(1) + sin(1)),$$

, $a = 1$. Then value number one for y is

$$y_0(x) = \phi(x) + L^{-1}(\sqrt(x) - x),$$

and the non linear part is

$$y_{n+1} = L^{-1}(A_n), n \geq 0,$$

now

$$A_0 = y_0^2,$$

$$A_1 = 2y_0y_1.$$

https://doi.org/10.54582/TSJ.2.2.21

The first term

$$y_0 = \frac{3031\sqrt{x}}{2340} - \frac{691\,x^{\frac{5}{2}}}{14040} - \frac{4\,x^3}{35} + \frac{691\,x^{\frac{9}{2}}}{280800} + \frac{16\,x^5}{3465} + \frac{1453\,x^{\frac{13}{2}}}{6739200} - \frac{64\,x^7}{675675} + \frac{7831\,x^{\frac{17}{2}}}{424569600} - \frac{823\,x^9}{51351300} \quad (5)$$

$$y_1 = \frac{-709642771867\sqrt{x}}{1593728136000} + \frac{709642771867\,x^{\frac{5}{2}}}{9562368816000} + \frac{1312423\,x^3}{6844500} - \frac{709642771867\,x^{\frac{9}{2}}}{191247376320000} - \frac{26221181\,x^5}{2032816500} - \frac{433\,x^{\frac{11}{2}}}{43875} + \frac{709642771867\,x^{\frac{13}{2}}}{8032389805440000} \quad (6)$$

from (5)and(6)get

$$y = \frac{1354711920533\sqrt{x}}{1593728136000} + \frac{239016215467\,x^{\frac{5}{2}}}{9562368816000} + \frac{3711361\,x^3}{47911500} - \frac{239016215467\,x^{\frac{9}{2}}}{191247376320000} - \frac{117841067\,x^5}{14229715500} - \frac{433\,x^{\frac{11}{2}}}{43875} + \frac{2441459958967\,x^{\frac{13}{2}}}{8032389805440000} - \frac{3824135347\,x^7}{113837724000} + \frac{7831\,x^{\frac{17}{2}}}{424569600} - \frac{823\,x^9}{51351300} \quad (7)$$

Table :The comparison between exact solution $y(x) = \sqrt{x}$ and ADM .

x	Exact	ADM	Absolut error
0.0	0	0	0000000000
0.1	0.3162277	0.268958	0.0472697
0.2	0.4472135	0.381205	0.0660085
0.3	0.5477225	0.468857	0.0111345
0.4	0.6324555	0.544868	0.0875875
0.5	0.7071067	0.61437	0.0927367
0.6	0.7745966	0.679837	0.0947596
0.7	0.8366600	0.742233	0.094427
0.8	0.8944271	0.801219	0.0932081
0.9	0.9486832	0.854974	0.0937092

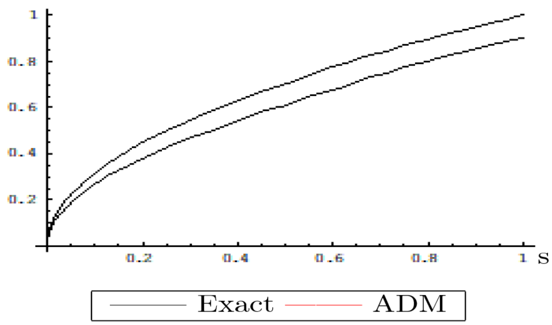

Figure 1: The exact solution $y = \sqrt{(x)}$ and the ADM solution $y = \sum_{n=0}^{2} y_n(x)$.

Example(2):

Consider the equation

$$y'' + \frac{1}{x}y' + (1 - \frac{1}{4x^2})y = e^x(2 + \frac{1}{x} - \frac{1}{4x^2}) + ln(y). \quad (8)$$

We use L^{-1} fot it and get

$$y = \phi(x) + L^{-1}(e^x(2 + \frac{1}{x} - \frac{1}{4x^2}) + L^{-1}(ln(y)),$$

and

$$\phi(x) = e\frac{sin(x)}{\sqrt{(x)}}(\frac{3}{2}cos(1) + sin(1)),$$

$a = 1$, then value number one for y is

$$y_0(x) = \phi(x) + L^{-1}(e^x(2 + \frac{1}{x} - \frac{1}{4x^2}) - x),$$

and the non linear part is

$$y_{n+1} = L^{-1}(A_n), n \geq 0,$$

now

$$A_0 = Ln(y_0),$$
$$A_1 = \frac{y_1}{y_0}.$$

The first term

$$y_0 = 1 + \frac{337\sqrt{x}}{1280} + x + \frac{x^2}{2} - \frac{337\,x^{\frac{5}{2}}}{7680} + \frac{11\,x^3}{210} + \frac{x^4}{24} + \frac{337\,x^{\frac{9}{2}}}{153600} + \frac{359\,x^5}{27720} \qquad (9)$$

$$y_1 = \frac{-2092108826092696642596689\sqrt{x}}{6502071962016153600000000} + \frac{3803982459842262082596689\,x^{\frac{5}{2}}}{3901243177209692160000000} + \frac{3163231\,x^3}{28672000} - \frac{1618149647\,x^{\frac{7}{2}}}{75497472000} + \frac{81265320071\,x^4}{18790481920000}$$

(10)

from (9)and(10)get

$$y = 1 - \frac{3802351923431312025966 89\sqrt{x}}{6502071962016153600000000} + x + \frac{x^2}{2} + \frac{2092108826092696642596689\,x^{\frac{5}{2}}}{3901243177209692160000000} + \frac{13995293\,x^3}{86016000} - \frac{1618149647\,x^{\frac{7}{2}}}{75497472000} + \frac{2592606200213\,x^4}{56371445760000} \qquad (11)$$

Table :The comparison between exact solution $y(x) = e^x$ and ADM .

x	Exact	ADM	Absolut error
0.0	1	1	0.000000000
0.1	1.10517	1.08684	0.1833
0.2	1.2214	1.19611	1.02529
0.3	1.34986	1.32006	0.0298
0.4	1.49182	1.45916	0.03266
0.5	1.64872	1.61445	0.03427
0.6	1.82212	1,78718	0.03494
0.7	2.01375	1.97876	0.03499
0.8	2.22554	2.19072	0.03482
0.9	2.4596	2.42469	0.03491

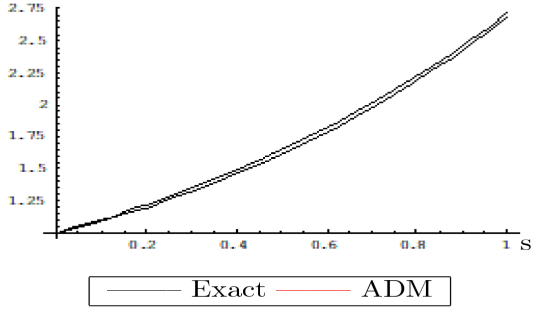

Figure 2: The exact solution $y = e^x$ and the ADM solution $y = \sum_{n=0}^{2} y_n(x)$.

3 Conclusion:

Adomian Decompostion Method is used to solve second orderd ordinary differential equation. The equation is Bessel's equation with new differential operator.

References

[1] D. L. Powers, Boundary value problems, 26-30 ,(2006).

[2] G. Adomian , A Review of the Decomposition Method in Applied Mathematics, Journal of Mathematical Analysis and Applications, 1988, 135, 501-544.

[3] G. Adomain and R. Rach: Inversion of nonlinear stochastic operators, J. Math. Anal. Appl., 91 (1983), 39-46.

[4] G. Adomain, Solving Frontier problems of Physics: The Decomposition Method, Kluwer academic publishers, London,1994.

[5] J. Biazar and K. Hosseini: A modified Adomain decomposition method for singular initial value Emden Fowrch, 5 (2016) 69-72.

[6] J. Lebl, Differential equations for engineers, 20, october, 2014.

[7] M. Epelde and J. Rivas : Bessel Function and Equations of Mathematical physics, 25, june (2015).

[8] M. Ston and p. M. Golgbart, : Mathematics for Physics, 2008.

[9] R. C. Daileda, An Introduction to Bessel Functions, 25 march, (2014).

[10] Y. Q. Hasan and L. M. Zhu: Modified Adomain decomposition method for singular initial value problems in the second-order ordinary differential equations, Surveys in mathematics and its applications, 3 (2008) 183193.